# Plasticity and stress heterogeneity influence on mechanical stress relaxation residual stress measurements

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**Abstract.** Two common problems of mechanical strain relaxation (MSR) residual stress measurement methods are investigated in this work: (1) assumption of stress uniformity and (2) the effect of plasticity at relaxation. A new MSR technique, designed specifically for highly non-uniform in-plane residual stress fields, is applied in this work to measure the residual stress field resulted from pure bending of an Al7075 alloy. The method involves introducing a straight cut across the whole part in a single increment, and collecting full field displacement fields from the side surface. Application of a 2D high resolution digital image correlation (DIC) method proved successful in this work. The reconstructed residual stress agrees well with that predicted by FE modelling. It is shown that the direction of the propagation of the slit has a major influence on plastic flow during relaxation. The major conclusion from this work is that it is possible to substantially reduce, or completely eliminate, plastic flow on relaxation by careful planning of the slit orientation and the cutting schedule.

## Mechanical stress relaxation basics revisited

Most, if not all, mechanical stress relaxation (MSR) methods rely on an elastic model to convert the measured relaxation strain or displacements into stresses. Examples of MSR techniques are: slitting and elastic crack solutions [1], hole drilling and a 2D elastic solution for a hole in a plane under a constant traction at infinity [2, 3], a hole boring and the Lamé's solution for a thick-walled cylinder [4] or an arbitrary (though usually planar) 3D cut with a corresponding elastic solution (the contour method) [5]. The relaxation can be measured with strain gauges, with non-contact optical methods, such as a laser speckle interferometry or a digital image correlation (DIC), which measure displacements, or with grid, shadow or photoelastic coating methods, which measure some functions of displacement or strain.

Although this general scheme has been used with great success over many decades to measure residual stresses in a variety of components, its success and applicability critically depend on two key assumptions. The relaxation process is assumed to be purely elastic. This is known not to be true in many practical applications, in particular, when the residual stress state is triaxial or when the magnitude of residual stress is close to yield, i.e. the most important cases of residual stress analysis. Although various plasticity corrections have been proposed, the issue is far from being completely resolved [3, 6, 7, 8].

Moreover, the vast majority of analytical models include assumptions on the residual stress state itself. Most models assume that the stress state is uniform in the immediate vicinity of the relaxation measurement location. For example, for the hole drilling method, the typical analytical model is a hole in a 2D plate under uniform far field stress. Or, in the case of a deep hole drilling method [3], stress is assumed constant in plane normal to the axis of the cut, but is allowed to change with depth. The contour method is one of the very few techniques which do not have any assumptions on the uniformity of the stress state [5]. However, the contour method suffers from inability, even in principle, to measure shear stresses.

These two problems - assumptions of stress uniformity and of purely elastic relaxation, are the focus of this work. To remove the stress uniformity assumptions, we have chosen a 2D elastic model suitable for an arbitrary self-equilibrated stress field at the end of a semi-infinite strip. It is shown in this work, that by carefully choosing the orientation and the direction of the propagation of the cut, plastic flow on relaxation can be minimised, thus significantly lowering experimental error.

## The analytical model

The Mathieu series solution for a self-equilibrated loading at the free end of a semi-infinite 2D strip is used in this work [9, pp. 61-62]. The solution is applied to the measured in-plane relaxation displacements. A more general application of the Mathieu's solution, suitable for out-of-plane measured displacements or photo-elastic measurements, is described in [10]. The solution uses the following even, $f$, and odd, $g$, stress functions:

$$f = e^{-\gamma x_1/c}\left(\xi \cos\frac{\gamma x_2}{c} + \frac{\gamma x_2}{c}\sin\frac{\gamma x_2}{c}\right); \qquad g = e^{-\phi x_1/c}\left(\psi \sin\frac{\phi x_2}{c} + \frac{\phi x_2}{c}\cos\frac{\phi x_2}{c}\right) \tag{1}$$

where $2c$ is the strip width, see Fig. 1(a), $x_1$ is along the strip, $x_2$ is normal to the strip axis. The boundary conditions are: $\sigma_{22} = \sigma_{12} = 0$ at $x_2 = \pm c$; $\int_{-c}^{c}\sigma_{11}\mathrm{d}x_2 = \int_{-c}^{c}\sigma_{12}\mathrm{d}x_2 = 0$ at $x_1 = 0$. The boundary conditions are satisfied if $f = \frac{\partial f}{\partial x_2} = 0$ at $x_2 = \pm c$, which leads to the following equations for the unknown dimensionless parameters of the stress functions, $\gamma, \xi, \phi, \psi$:

$$\sin 2\gamma + 2\gamma = 0; \qquad \xi = -\gamma \tan\gamma; \qquad \sin 2\phi - 2\phi = 0; \qquad \psi = -\phi/\tan\phi \tag{2}$$

which have infinite number of solutions. The first non-zero roots are: $\gamma_1 = 2.106$, $\xi_1 = 1.125$, $\phi_1 = 3.749$, $\psi_1 = 1.384$. Combining the even and the odd stress functions, we construct the infinite series representation of the stress function, suitable for any arbitrary self-equilibrated loading at the $x_1 = 0$ boundary:

$$\theta = \sum_{i=1}^{\infty} a_i\Re(f_i) + b_i\Im(f_i) + c_i\Re(g_i) + d_i\Im(g_i) \tag{3}$$

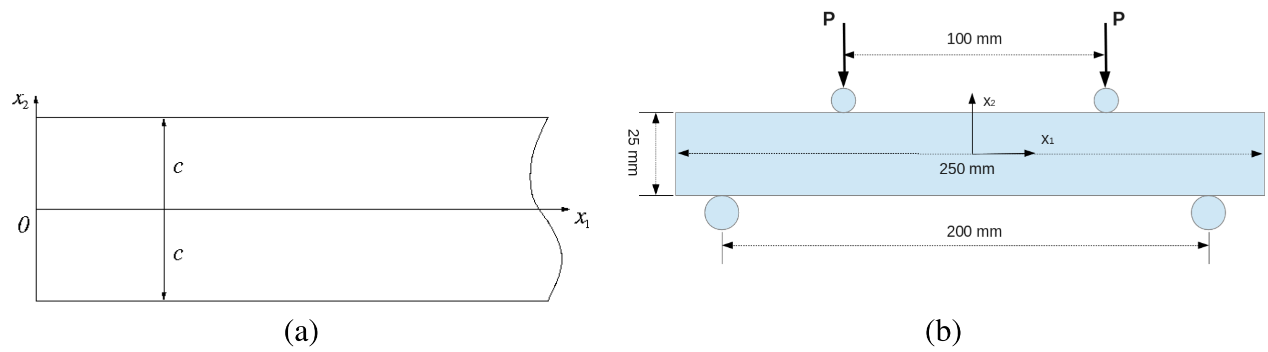



Fig. 1: (a) Schematic of the problem geometry for a 2D semi-infinite rectangular strip, of width $2c$, with an arbitrary self-equilibrated end load and (b) schematic diagram of a four-point bend test, used in this work, for controlled residual stress creation. The specimen is cut with wire EDM on $x_1 = 0$ symmetry plane.

Finally, using the Hooke's law for the plain stress case, we can represent the measured in-plane displacement at any point on the surface via functions of the unknown coefficients, $a_i, b_i, c_i, d_i, i = 1, 2, \ldots, \infty$, and several position dependent integrals:

$$u_1 = \frac{1}{E}\int_{-\infty}^{0}\left(\frac{\partial^2\theta}{\partial x_2^2} - \nu\frac{\partial^2\theta}{\partial x_1^2}\right)\mathrm{d}x_1; \quad u_2 = \frac{1}{E}\int_{-c}^{c}\left(\frac{\partial^2\theta}{\partial x_1^2} - \nu\frac{\partial^2\theta}{\partial x_2^2}\right)\mathrm{d}x_2 + u_2(x_1, 0) \tag{4}$$

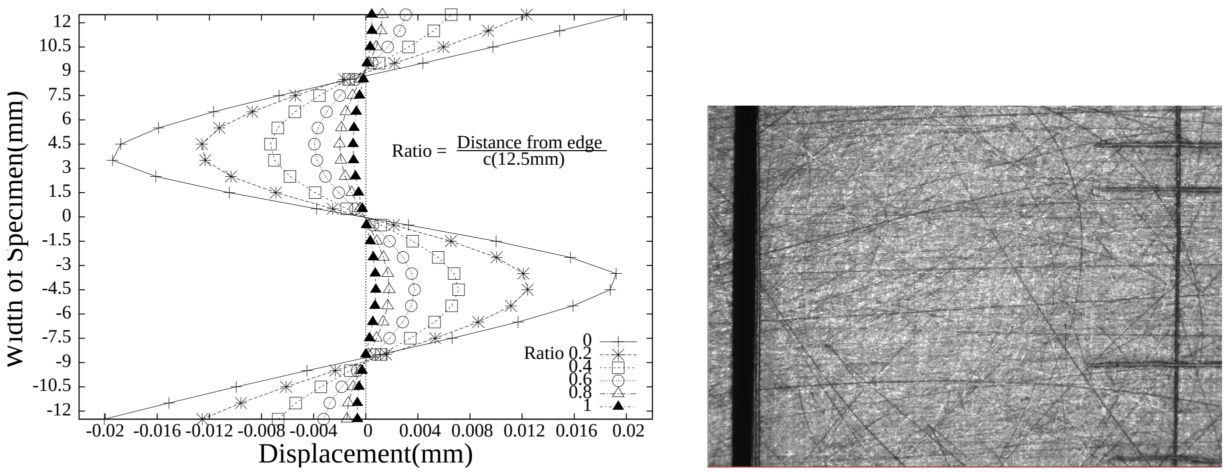
If the series in Eqn. (3) are truncated at $n$, then the problem is that of minimising some norm of $\mathsf{Ax}$-$\mathsf{B}$, where $\mathsf{x}$ is the vector of $4n$ unknown coefficients, $\mathsf{B}$ is the vector of $2m$ measured in-plane relaxation displacements (two displacements, $u_1$ and $u_2$ are measured at each point), and $\mathsf{A}_{2m\times 4n}$ is the matrix of integral coefficients, which are only functions of position. If $m \gg n$, then the problem is easily solved, e.g. in a linear least squares sense. Finally, the stresses at $x_1 = 0$ are calculated from the stress function, $\theta$.

Numerical verification of this method is given in [10, 11]. In this work, we are interested in a thorough experimental validation.

## Experiments

Since the analytical model is two-dimensional, the stress state through thickness, along $x_3$, is assumed constant. In addition, our experience shows that $m/n > 10^2$ is required for the stability of the method. Hence, in practice, the method requires some full field measurement technique, such as a laser speckle interferometry or DIC. In this work DIC is used, because of its low cost and simplicity.

Experiments were performed using 25×25×250 mm bars machined of Aluminium 7075-T6 ($\sigma_Y$= 506 MPa, $E$=71.7 GPa), in which the residual stress field was introduced by 4-point bending along $x_2$, see Fig. 1b. After unloading, the specimens were cut on the symmetry plane, $x_1 = 0$, longitudinal midsection of the bar, with ⌀0.1 mm wire EDM.




(a) (b)

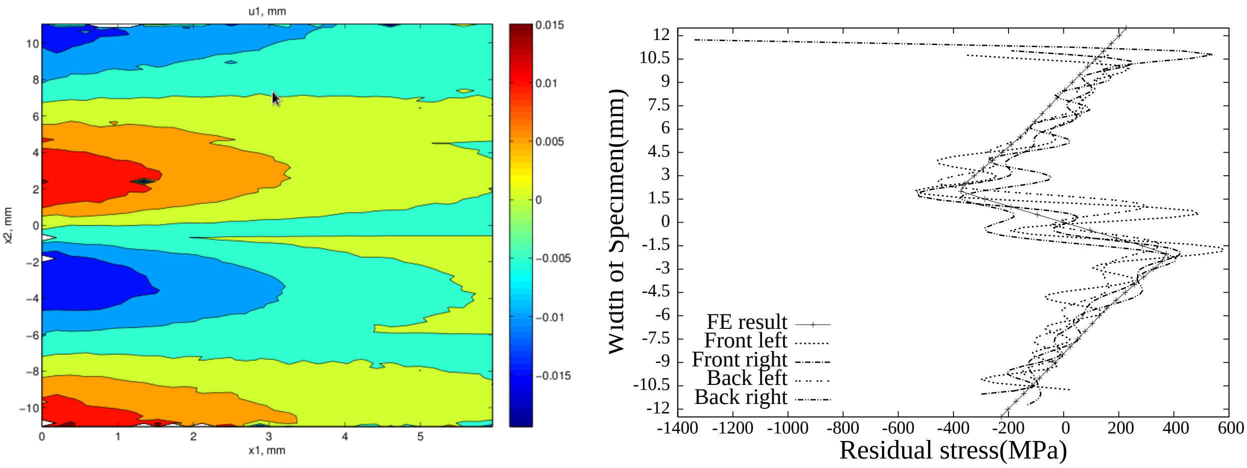
Fig. 2: (a) FE simulated relaxation displacements for a 25×25 cross section Al7075-T6 four-point bend bar at different distances from the cutting edge for 9 mm preload displacement and (b) specimen surface prepared for DIC using P180 sandpaper. The image captures about 10.6×8 mm surface fragment.

Preliminary FE studies of relaxation were conducted to help estimate the required resolution of the displacement measurement technique. Fig. 2a shows that, as predicted by Eqn. (1), relaxation displacements decay exponentially away from the cut. Hence, it is critical to be able to collect data as close to the cut as possible. However, displacements in excess of 12 $\mu$m are expected at distances of $< 0.2c$ from the cut. The measurement system was put together with the aim to resolve displacements at least down to 12 $\mu$m. Using of a 3D digital image correlation optical system is technically complicated at high magnification, mainly due to practical challenges of fitting two cameras, lenses, and the light source very close to the specimen surface. Hence, in this work, we opted for a 2D DIC system. The system consisted of a 10-bit 1392×1040 pixel CCD, fixed focus optics with a working distance of 175 mm and 7.6 $\mu$m spatial resolution, a ring light illumination and the Istra 4D DIC software [12].

The optimal surface preparation in this work was achieved from light scratching of the aluminium surface with P180 sand paper, see Fig. 2b. This optical system produced images of about 10.6×8 mm of the specimen surface. Four overlapping images were recorded along the cut and analysed separately.

The model assumes that $u_1(x_1 \to +\infty) \to 0$, hence, it is necessary to remove the rigid body motions from the analysis. The in-plane translation and rotation can be compensated for in the DIC software itself. The compensation for the out-of-plane motion within the 2D DIC framework is impossible, because it is treated as extra in-plane displacement. Hence this has to be done as a post-processing step. In this work, we followed [13]. The four stitched displacement images, produced in one of the experiments, are shown in Fig. 3a.

The reconstructed residual stress shows good agreement with that predicted by the FE, see Fig. 3b. Previously, similarly good residual stress agreement between the FE prediction and the measurements was achieved for Al2024-T4 [11].




(a) (b)

Fig. 3: (a) The measured horizontal displacement field, $u_1$ (mm). The DIC subset size was 25×25 pixels with no overlap. Note a horisontal discontinuity line roughly in the middle right of the image. This indicates an imperfect stitching between images 2 and 3, perhaps due to imperfect rigid body compensation. Also note few white spots at the edges where DIC failed. Application of DIC at the image edges is always complicated by the fact that some material information is present in one image, but not in the other, e.g. a bit of material surface moves into the field of view after the cut, that was not there before the cut. (b) Four DIC experiments on the same Al 7075-T6 specimen, i.e. a single cut analysed from 4 resulting surfaces. Note that a good overall agreement is disturbed by oscillations in the series representation.

## Plastic flow on cutting

As the cut progresses, residual stress redistributes in such a way that the force and momentum equilibrium are always satisfied. Although slitting, whether mechanical or wire EDM, always causes local plastic deformation in the immediate vicinity of the cutting edge, of greater concern is global plastic flow, affecting a significant volume of material, potentially far from the cutting edge. This non-local plastic flow has been observed experimentally and predicted numerically, see e.g. [3, 6, 7, 5, 8], particularly when the magnitude of residual stress is close to yield and when the stress state is strongly triaxial.

For the slitting method in particular, the non-local plastic flow can be significant if the stresses on the top and bottom of the beam ($x_2 = \pm$ c) are of different sign [6]. As the material, and the stresses, are removed on top face, the stress on the bottom face must increase to preserve the momentum equilibrium. An additional potential problem in the proposed method is that the relaxation displacements must be measured as close to the cut as possible, because they decay very quickly away from it. This means that even local plastic flow can cause errors in the measured stresses, because larger displacements can only be interpreted by the elastic model as resulting from higher elastic stresses.

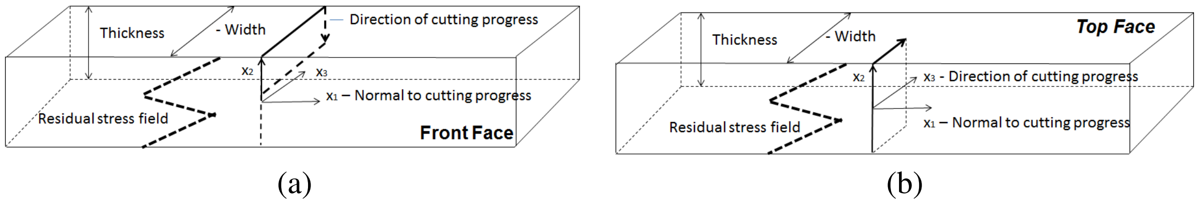



Fig. 4: Two cutting schedules for a prismatic bar analysed in this work, with the residual stress produced by a four-point bending, showing: (a) The conventional slitting method, e.g. from [6], representing the wire advancing along $x_2$, from the top face towards the bottom face. The stress along the cutting edge is constant. (b) Slitting along $x_3$, from the front face to the rear face. The stress along the cutting edge is non-uniform.

To measure a residual stress field, resulting from bending about $x_3$, the conventional slitting method introduces the slot along $x_2$, the thickness direction, see Fig. 4a. Because each slit increment removes a constant stress field along the cutting edge, a non-zero residual force is removed, causing the residual stress to increase on the bottom surface, potentially leading to plastic flow if the stresses are already close to yield. In contrast, in the proposed method the cut can be introduced along $x_2$ or along $x_3$, the width direction, see Fig. 4b. When the slot is advanced along $x_3$, it cuts a layer of material with non-uniform, self-equilibrated stress profile along the cutting edge. Hence, a zero residual force is removed, and no redistribution of residual stress is caused. The end result is the same in both cases: the specimen is cut on the $x_1 = 0$ symmetry plane. However, the two cutting schedules differ in the amount of plastic flow they induce.

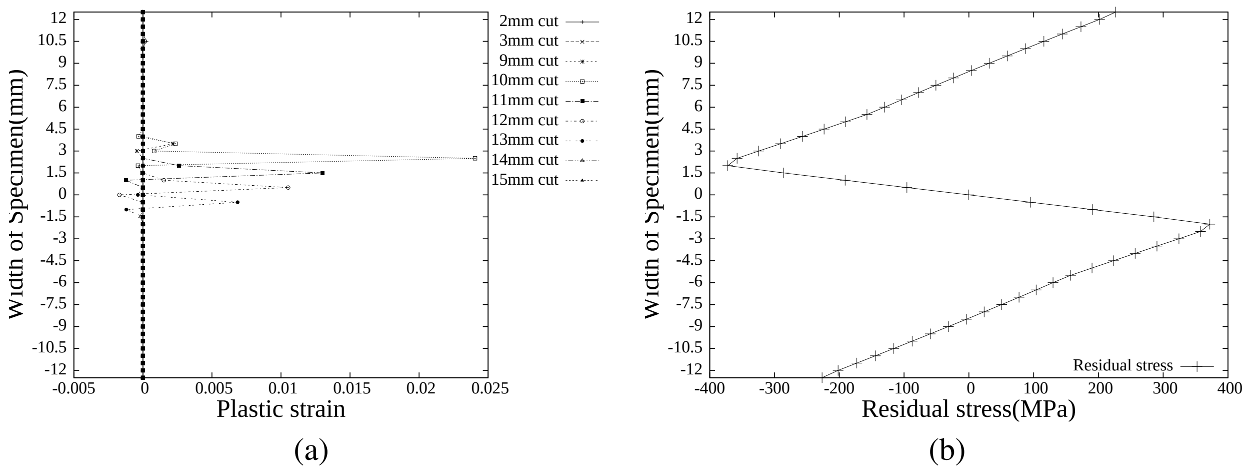



Fig. 5: Plastic flow and residual stress field are measured along the cut ($x_1$ = 0) when advancing the cut along $x_2$, i.e. through thickness, from top to bottom, showing: (a) Equivalent plastic strain profile through thickness between 2 consecutive cutting increments of 1 mm depth only. Note that some cutting increments produce very little or no plastic flow and those are removed from this graph. However, at cut depths of between 9 and 13 mm, corresponding to $x_2$ between -1.5 and 4.5mm, the plastic flow is substantial. This region, of course, matches the region of the residual stress peak, as shown in (b). The peak residual stress to yield ratio is 0.75 in this example.

To test this hypothesis further, a detailed FE study of plastic flow on cutting was conducted. The residual stress field was generated with the peak stress to yield ratio of 0.75, see Fig. 5b. It was found that when the slot is advanced along $x_2$, from the top face towards the bottom face, there is substantial stress redistribution that causes plastic strain of over 2% in the region of the topmost residual stress peak, see Fig. 5. However, when the wire is advanced along $x_3$, from the front to the rear face, there is no detectable plastic flow on the front face. Thus, the FE study quantitatively confirms our hypothesis on the influence of the direction of the propagation of the slot on the induced plastic flow. This observation is critical in choosing the optimal cutting schedule to reduce plastic flow.

## Discussion, conclusions and future work

A simple observation from the plastic flow study, that different cutting schedules, which produce the same cut surface, might cause different amounts of plastic flow, has wide implications. It means that the cut must match not only the chosen analytical model, but also, in some way, the expected residual stress field. In other words, if there is some prior knowledge of the expected residual stress, then methods based on different cutting schedules might have different success.

In this work, a cut is progressed in such as way, as to remove the layer of material with self-equilibrated non-uniform stress. This means that as the slot advances, the residual force that is removed is always zero. Hence the only stress redistribution that is possible with such cut is a universal stress reduction across the whole remaining ligament ahead of the cutting edge. In this case, the use of the measured surface displacements within a purely elastic 2D analytical model yields a good residual stress estimate.

Various technical issues need to be resolved to reduce the experimental error in the proposed method. In particular, a better understanding of the stability of the analytical model is required, i.e. the relationship between the number of terms in the series representation of the stress function and the number and locations of the data points. The DIC method is inherently subjective. Larger subset sizes lead to more averaging, i.e. more accurate results on average, at the expense of not resolving sharp gradients, while smaller subset sizes lower the accuracy and increases the noise, while allowing for the resolution of higher gradients. The question of the optimal subset size must be investigated. Mismatch between individual images with fragments of the specimen surface along the cut needs to be reduced. It is expected that this can be achieved if the rigid body correction is performed on all images together, rather than separately for each pair of DIC images.

However, the major direction for future research is an assessment of the sensitivity of other MSR methods, such as the contour or the deep hole drilling, to the direction and the manner of propagation of the cut.

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