Measuring locally non-uniform in-plane residual stress with straight cuts and DIC

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Many diverse destructive residual stress measurement methods exist, however all of them involve the same four general steps: (1) choosing a suitable *elastic* model, (2) cutting, (3) measuring relaxation and (4) calculating stresses using the elastic model with the measured relaxation [1]. For example, the hole drilling method uses (1) the 2D elastic solution for a circular hole in an infinite plate under uniform loading at infinity, (2) circular hole drilling, (3) typically 3 strain gauges located at certain positions around the hole, and (4) generalised Hooke's law in 2D to convert two normal and one shear strain into stresses [2].

One of the major limitations of residual stress methods is due to the limitations (or assumptions) built into the solution of the corresponding elastic problem. For example, the hole drilling methods, both 2D and 3D [3], use elastic solution that assumes constant in-plane stress/strain fields within 3-5 mm hole radius from the hole axis. When these assumptions are invalid, e.g. across welds, these methods cannot be used.

The use of the Mathieu series solution for a semi-infinite strip with arbitrary self-equilibrated loading at the end [4, p.61], see Fig. 1(a) for the measurement of residual stresses was proposed in [5]. At the heart of the method is the problem of minimising the norm:

$$\min_x ||\mathbf{A}x - u||_2 \tag{1}$$

where u is a vector of $2m$ displacement values, $\mathbf{A}$ is a $2m \times n$ constant matrix, dependent only on the problem geometry, and x is the vector of n unknown coefficients for the terms in the Mathieu series. To minimise the influence of experimental errors, typically $m \gg n$.

In this work we apply the digital image correlation (DIC) method to measure the relaxation displacements. The DIC is attractive because it is relatively cheap, easy and can be applied in field conditions [6]. All that is required is to apply a suitable pattern, record one image prior to cutting, and the second image after the cutting. Then, assuming the surface pattern is preserved during the cutting, the DIC analysis will produce the displacement data for use in Eqn. (1).

The behaviour of the Mathieu solution was first studied with a finite element (FE) method. Relaxation displacements were simulated by those caused by applied self-equilibrated loading on the boundary. Then the FE calculated displacements were used in Eqn. (1) to reconstruct the stresses. This worked well, even for a discontinuous applied stress field, see Fig. 2(a).

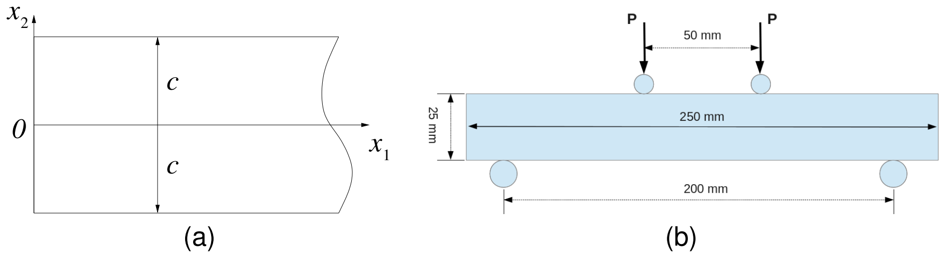
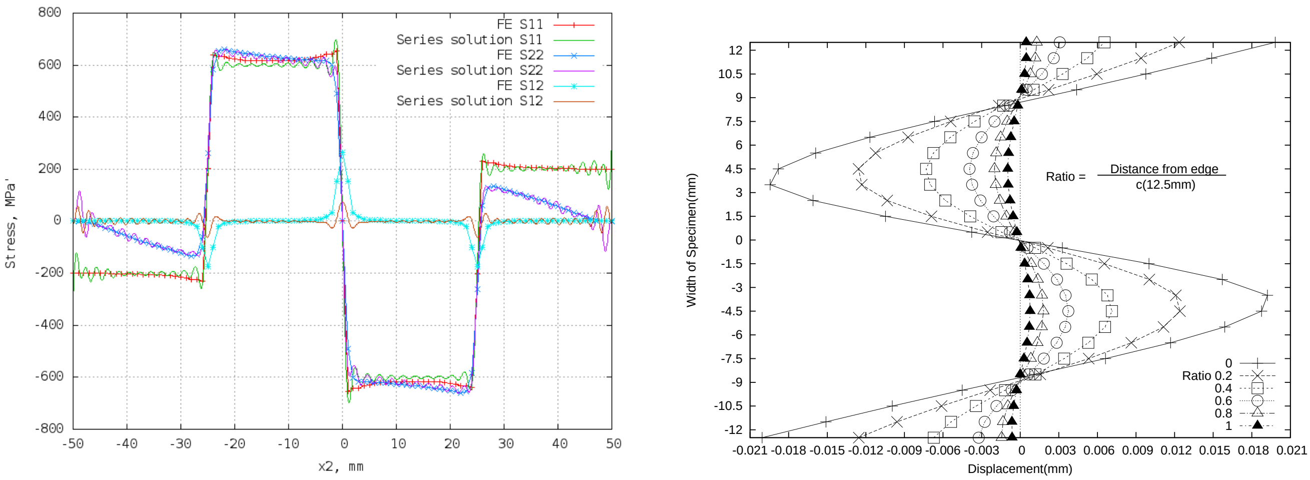


Figure 1: Mathieu series solution for a semi-infinite rectangular strip, of width $2c$, with an arbitrary self-equilibrating end load, showing (a) schematic of the problem geometry. The boundary conditions are: $x_2 = \pm c : \sigma_{22} = \sigma_{12} = 0$ and $x_1 = 0 : \int_{-c}^{c} \sigma_{22} \mathrm{d}x_2 = \int_{-c}^{c} \sigma_{12} \mathrm{d}x_2 = 0$; and (b) schematic diagram of a 4-point bend test for controlled residual stress creation, the symmetry plane is $x_1 = 0$.

The experiments involve generating residual stresses in 25×25×250mm (c=12.5mm) aluminium 7075-T6 bars (σ_Y=503MPa, UTS=572MPa) in a 4-point bend test, see Fig. 1(b), followed by EDM cutting along the symmetry plane. A major problem is that of pattern preservation during EDM cutting.

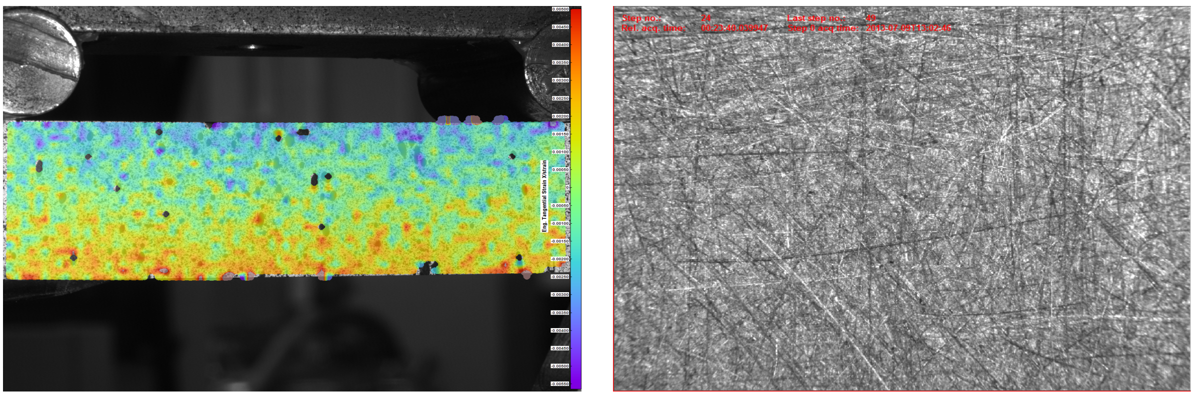
Relaxation displacements decay exponentially away from the cut. Fig. 2(b) shows the relaxation analysis for our specimen. The plots are displacements along x_1 at lines $x_1 = 0$ to $x_1 = c$. The maximum displacement, at the boundary, is 20μm. At $x_1 = 0.2c$ the maximum displacement drops to about 12μm, with most of the data at less than 5μm. Hence it is critical to be able to collect the data as close to the cut as possible.



(a) (b)

Figure 2: (a) residual stress calculated from the displacement field ("series solution") due to anti-symmetric discontinuous *applied* boundary stress, simulated with finite element analysis ("FE"), $c = 50$mm; and (b) a relaxation analysis for a 4-point bend residual stress field generated in a 25×25mm cross section Al7075-T6 beam under 6mm vertical displacement, P=27.65kN, see Fig. 1(b).

High magnification is required to measure small displacements, Fig. 3. However, with high magnification, only a small fragment of the specimen surface can be recorded. There is a balance between the required magnification to achieve the necessary spatial resolution and the increase in the number of images required to record the whole of the displacement field along the boundary. Our current optical system gives 7.6μm/pixel spatial resolution. With sub-pixel resolution we can measure displacements of 0.8μm, if a suitable pattern is found, Fig. 3(b).



(a) (b)

Figure 3: (a) A 4-point bend DIC strain field with a spray pattern, showing very high level of noise at 60μm/pixel spatial resolution; and (b)High magnification Image. Scrach pattern is obtained from P180 sandpaper. A scratch pattern at 7.6μm/pixel spatial resolution(11 × 8mm size of field of view).

References

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