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Measuring locally non-uniform in-plane residual stress with a straight cut and DIC

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Outline

Problem: uniform stress fields in analytical model

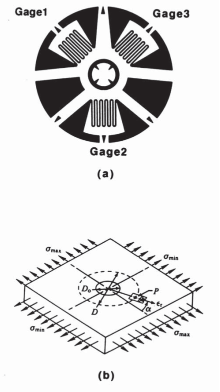
New: non-uniform in-plane stress analytical model

Experiments: 4-point bending + DIC

Results

Conclusions

- All destructive methods consist of 4 steps:
  1. analytical model
  2. destruction
  3. relaxation measurement
  4. solution of inverse problem
- E.g. – hole drilling model: constant stress field for 3-5 diameters
- Real res. stresses are not uniform!



Hole-drilling method for measuring residual stresses. (a) Typical three-element strain-gage rosette. (b) In-plane strain components caused by release of residual stresses through the introduction of a hole. Source: ASTM E 837-92. From Testing of Coatings, Walter Riggs, Tubal-Cain Co. Inc., and Ken Couch, Protech Lab Corp., as published in *Handbook of Thermal Spray Technology*, J.R. Davis (Ed.), ASM International, p 271, 2005.

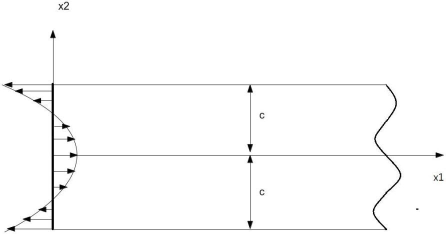
- Semi-infinite 2D strip of width $2c$
- Self-equilibrated loading at $x_1 = 0$:

$$\int_{-c}^{c} \sigma_{11} \mathrm{d}x_2 = \int_{-c}^{c} \sigma_{12} \mathrm{d}x_2 = 0$$

- Free boundaries:

$$x_2 = \pm c : \sigma_{22} = \sigma_{12} = 0$$

- Mathieu series solution



- Even, f, and odd, g, analytical stress functions:

$$f = Ce^{-\gamma x_1/c}\left(\xi \cos\frac{\gamma x_2}{c} + \frac{\gamma x_2}{c}\sin\frac{\gamma x_2}{c}\right)$$
$$g = Ce^{-\phi x_1/c}\left(\psi \sin\frac{\phi x_2}{c} + \frac{\phi x_2}{c}\cos\frac{\phi x_2}{c}\right)$$

- From BC there are infinite number of solutions for γ, ξ, ϕ, ψ:

$$\sin 2\gamma + 2\gamma = 0 \quad ; \quad \xi = -\gamma\tan\gamma$$
$$\sin 2\phi - 2\phi = 0 \quad ; \quad \psi = -\phi/\tan\phi$$

- The general solution stress function:

$$\theta = \sum_{i=1}^{\infty} a_i \mathrm{Re} f_i + b_i \mathrm{Im} f_i + c_i \mathrm{Re} g_i + d_i \mathrm{Im} g_i$$

Integrating Hooke's law [1]:

$$Eu_1 = \sum_{i=1}^{\infty} a_i \int \mathrm{Re}\left(\frac{\partial^2 f_i}{\partial x_2^2} - \nu \frac{\partial^2 f_i}{\partial x_1^2}\right) \mathrm{d}x_1 + b_i \int \mathrm{Im}\left(\frac{\partial^2 f_i}{\partial x_2^2} - \nu \frac{\partial^2 f_i}{\partial x_1^2}\right) \mathrm{d}x_1$$

$$+c_i \int \mathrm{Re}\left(\frac{\partial^2 g_i}{\partial x_2^2} - \nu \frac{\partial^2 g_i}{\partial x_1^2}\right) \mathrm{d}x_1 + d_i \int \mathrm{Im}\left(\frac{\partial^2 g_i}{\partial x_2^2} - \nu \frac{\partial^2 g_i}{\partial x_1^2}\right) \mathrm{d}x_1$$

$$Eu_2 = \sum_{i=1}^{\infty} a_i \int \mathrm{Re}\left(\frac{\partial^2 f_i}{\partial x_1^2} - \nu \frac{\partial^2 f_i}{\partial x_2^2}\right) \mathrm{d}x_2 + b_i \int \mathrm{Im}\left(\frac{\partial^2 f_i}{\partial x_1^2} - \nu \frac{\partial^2 f_i}{\partial x_2^2}\right) \mathrm{d}x_2$$

$$+c_i \int \mathrm{Re}\left(\frac{\partial^2 g_i}{\partial x_1^2} - \nu \frac{\partial^2 g_i}{\partial x_2^2}\right) \mathrm{d}x_2 + d_i \int \mathrm{Im}\left(\frac{\partial^2 g_i}{\partial x_1^2} - \nu \frac{\partial^2 g_i}{\partial x_2^2}\right) \mathrm{d}x_2$$

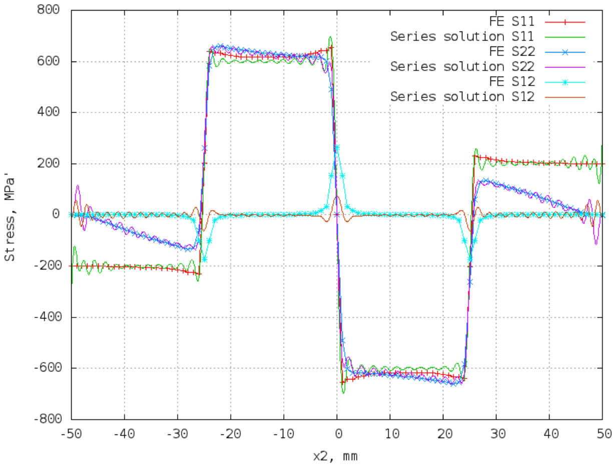
Finally get the LLS problem:

$$\min_x ||\mathbf{A}x - u||_2$$

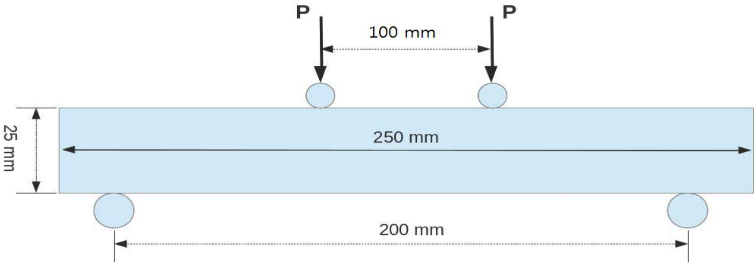
- $x = (a_1, b_1, c_1, d_1, \dots a_N, b_N, c_N, d_N)^T$ is the vector of unknown coeff., $4N$ long, where N is the number of terms
- u is the vector of measured displacements, $2M$ long, for M exp. points
- As always for the stability of LLS, $M \gg N$
- A is a matrix of integral functions of f and g taken at the locations of the measurement points.

FE validation

- FE ⟶ Apply Stresses ⟶ Disp ⟶ Inverse method ⟶ Stress out
- Works even with discontinuities !!



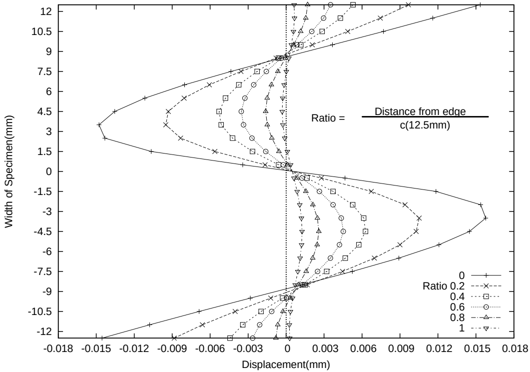
Four-point beding of Al2024-T4 25 × 25mm bar



Material	Yield Stress (Mpa)	Modulus of Elasticity (Mpa)	UTS (MPa)	Poisson's Ratio	Strength Coefficient(MPa)	Strain Hardening Exponent
Al 2024-T4	346	73120	490	0.33	806	0.2

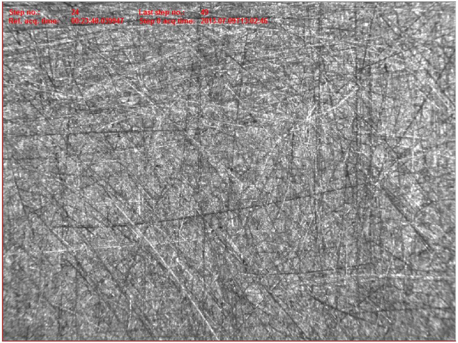
Is relaxation measurable?

- 25 × 25 mm bar under 8mm vertical displacement, P= 40.6KN.
- The maximum displacement, at the edge, around 15 μm
- Decays very quickly! At 0.4*c* away from the edge, only 5 μm.



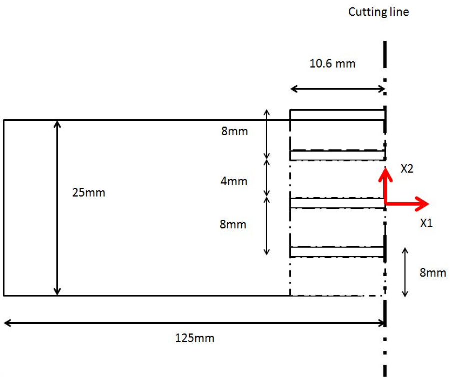
Optical system and pattern

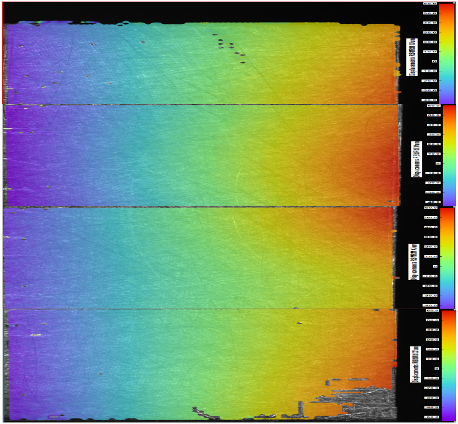
- 7.6μm/pixel spatial resolution.
- Field of view is 10.6 × 8mm
- 175 mm working distance.
- P180 sandpaper
- Ring light
- 10bit CCD with a 1392 × 1040



DIC Results

Subset and stepsize are 17 ×17 pixels





Out of plane translation effect

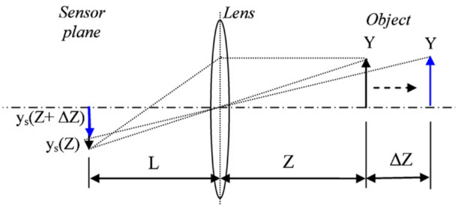
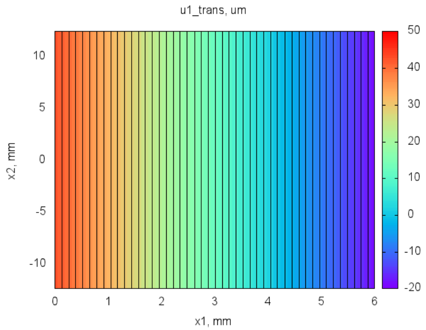


Figure: *

Effect of out-of-plane translation [2]

$$U(\Delta Z) = x_s(Z+\Delta Z) - x_s(Z) = x_s\left(-\frac{\Delta Z}{Z}\right)$$



3mm towards the camera !!

Reconstructed residual stress

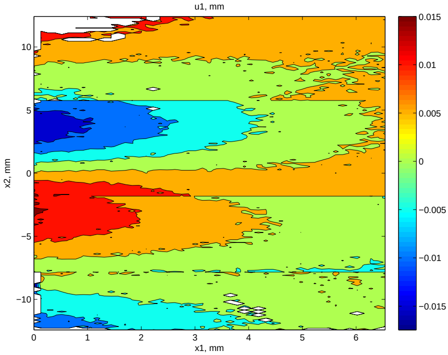


Figure: *

u_1 compensated for out of plane motion.

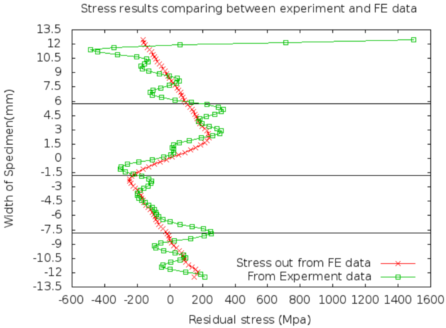


Figure: *

Calculated residual stress.

Problems

- Pattern preservation
- Displacement resolution.
- high magnification = many images – complex experiment!
- Very fine pattern is required to achieve good correlation.

Conclusions

- It works!
- Problems were
  1. Out-of-plane motion is an issue
  2. Suitable pattern choice and preservation
  3. Need more work to get better accuracy of the measured displacements.
- To do:
  1. Explore how to improve the accuracy of the measured res.stress and whether the method can also be applied to other res.stress problems
  2. Further away, measuring res.stress in heterogeneous materals, e.q. composites.

Thank you for listening

ANY QUESTIONS?

References

[1] I. A. Razumovskii and A. L. Shterenlikht. Determining the locally-nonuniform residual-stress fields in plane parts by the sectioning method. *Journal of Machinery Manufacture and Reliability C/C of Problemy Mashinostroeniia i Nadezhnosti Mashin*, 4:40–45, 2000.

[2] M A Sutton, J H Yan, V Tiwari, H W Schreier, and J J Orteu. The effect of out-of-plane motion on 2D and 3D digital image correlation measurements. *Optics and Lasers in Engineering*, 46:746–757, 2008.